A Preliminary Study of Space-time Finite Element Eddy-current Analysis

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A space-time finite element method is developed for the eddy-current analysis that can handle temporally-variant spatial finite elements arising in the analysis of moving objects. The vector potentials allocated on space-time finite elements naturally expresses the speed electromotive force. A Space-time triangular prism finite element is examined to represent the eddy-current field in space-time successfully.

Index Terms—**Eddy current, space-time finite element, speed electromotive force**

I. Introduction

The time is usually formulated independently of the space in the finite element electromagnetic field analysis. To obtain the time evolution of electromagnetic field, a sequential scheme is generally used in a step-by-step manner with a uniform time-step. However, it is possible to handle the space and time in a unified manner in the Maxwell equations. This means that the computational finite-elements can be generated in the space-time to handle temporally-variant spatial finite elements arising in the analysis of moving objects.

Previous studies [1]-[3] have successfully introduced a temporal convolution for the symmetric formulation of space-time finite-element (FE) eddy-current analysis. However, the temporal convolution was only applied to the temporally uniform space-time elements and its application to the analysis of moving objects is not straightforward.

This paper proposes a Galerkin type space-time FE analysis to develop general polygon-type finite elements such as space-time parallelepiped and prism elements, where the speed electromotive force is naturally introduced.

II. Space-Time Allocation of Vector Potential

For example, the vector potential $\boldsymbol{A}$ is allocated on the edges of parallelogram space-time element as in Fig. 1, where the two edges are slanted as $(v\Delta t, \Delta t)$. The

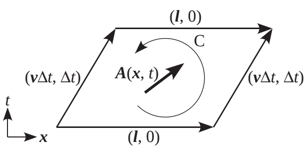

Fig.1 parallelogram space-time element

contour integral of $\boldsymbol{A}$ along the parallelogram is given as

$$\begin{aligned}\oint_C \boldsymbol{A}\cdot \mathrm{d}\boldsymbol{s} &= (\boldsymbol{v}\Delta t)\cdot[\boldsymbol{A}(\boldsymbol{x}+\frac{\boldsymbol{l}}{2},\ t)-\boldsymbol{A}(\boldsymbol{x}-\frac{\boldsymbol{l}}{2},\ t)]\\ &-\boldsymbol{l}\cdot[\boldsymbol{A}(\boldsymbol{x}+\frac{\boldsymbol{v}\Delta t}{2},\ t+\frac{\Delta t}{2})-\boldsymbol{A}(\boldsymbol{x}-\frac{\boldsymbol{v}\Delta t}{2},\ t-\frac{\Delta t}{2})]\\ &\approx \boldsymbol{v}\cdot(\boldsymbol{l}\cdot\boldsymbol{\nabla})\boldsymbol{A}\Delta t-\boldsymbol{l}\cdot[\frac{\partial \boldsymbol{A}}{\partial t}+(\boldsymbol{v}\cdot\boldsymbol{\nabla})\boldsymbol{A}]\Delta t\\ &=[\boldsymbol{v}\cdot(\boldsymbol{l}\cdot\boldsymbol{\nabla})-\boldsymbol{l}\cdot(\boldsymbol{v}\cdot\boldsymbol{\nabla})]\boldsymbol{A}\Delta t-\Delta t\ \boldsymbol{l}\cdot\frac{\partial \boldsymbol{A}}{\partial t}\\ &=[(\boldsymbol{l}\times\boldsymbol{v})\cdot\ \mathrm{curl}\boldsymbol{A}-\boldsymbol{l}\cdot\frac{\partial \boldsymbol{A}}{\partial t}]\Delta t\\ &=\boldsymbol{l}\cdot[\boldsymbol{v}\times\mathrm{curl}\boldsymbol{A}-\frac{\partial \boldsymbol{A}}{\partial t}]\Delta t\\ &=\boldsymbol{l}\cdot(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B})\Delta t \end{aligned} \tag{1}$$

Thus, the speed electromotive force is naturally introduced, where the ϕ=0 gauge is used for simplicity. In the 4D space-time, the electromagnetic four-potential $(A, \phi/c)$ is defined. The contour integral of four-potential is similarly yields the speed electromotive force.

III. Space-Time Finite Elements

Space-time finite elements can be constructed in a similar way to conventional spatial finite-elements. For example, the triangular prism element shown in Fig. 2 is examined in this study. The scalar interpolation function N_1 for the node 1 given as

$$N_1 = \frac{1}{2V}(a_1 + b_1 x + c_1 t)(y_2 - y) \tag{2}$$

where V is the volume of element; a_1, b_1 and c_1 are

$$a_1 = x_2 t_3 - x_3 t_2 \quad b_1 = t_2 - t_3 \quad c_1 = x_3 - x_2 \tag{3}$$

where x_i and t_i are coordinate of node i.

IV. Formulation of Space-Time Finite Element Method

This article examines xyt-3D space-time eddy-current

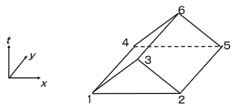

Fig.2 Triangular prism element

field described as

$$\frac{\partial}{\partial x}(\nu\frac{\partial A}{\partial x}) + \frac{\partial}{\partial y}(\nu\frac{\partial A}{\partial y}) = -J_0 + \sigma\frac{\partial A}{\partial t} \qquad (4)$$

where A is the z-component of vector potential, J_0 is the imposed current density and ν is the reluctivity. The Galerkin finite-element method (FEM) is used to derive the weak form. The interpolation function N_i is multiplied to (4) as the weighting function as

$$\iiint_\Omega (\frac{\partial}{\partial x}(\nu\frac{\partial A}{\partial x})N_i + \frac{\partial}{\partial y}(\nu\frac{\partial A}{\partial y})N_i)\mathrm{d}x\mathrm{d}y\mathrm{d}t$$
$$= \iiint_\Omega \sigma\frac{\partial A}{\partial t}N_i\mathrm{d}x\mathrm{d}y\mathrm{d}t - \iiint_\Omega J_0 N_i\mathrm{d}x\mathrm{d}y\mathrm{d}t\ . \qquad (5)$$

Integrating the left hand side by parts, (6) is obtained.

$$(\text{left hand side}) = \int\oint \nu N_i(\frac{\partial A}{\partial x}\mathrm{d}y - \frac{\partial A}{\partial y}\mathrm{d}x)\mathrm{d}t$$
$$- \iiint_\Omega (\nu\frac{\partial A}{\partial x}\frac{\partial N_i}{\partial x} + \nu\frac{\partial A}{\partial y}\frac{\partial N_i}{\partial y})\mathrm{d}x\mathrm{d}y\mathrm{d}t \qquad (6)$$

The natural boundary condition is assumed to eliminate this term. Thus, the weak form (7) is obtained.

$$\iiint_\Omega (\nu\frac{\partial A}{\partial x}\frac{\partial N_i}{\partial x} + \nu\frac{\partial A}{\partial y}\frac{\partial N_i}{\partial y} + \sigma\frac{\partial A}{\partial t}N_i)\mathrm{d}x\mathrm{d}y\mathrm{d}t$$
$$= \iiint_\Omega JN_i\mathrm{d}x\mathrm{d}y\mathrm{d}t \qquad (7)$$

The linear system of equations (8) is obtained from (7)

$$([K] + [C])\{A\} = \{F\} \qquad (8)$$

where

$$K_{ij} = \iiint_\Omega (\nu\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x} + \nu\frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y})\mathrm{d}x\mathrm{d}y\mathrm{d}t \qquad (9)$$

$$C_{ij} = \iiint_\Omega (\sigma N_i\frac{\partial N_j}{\partial t})\mathrm{d}x\mathrm{d}y\mathrm{d}t \qquad (10)$$

$$F_i = \iiint_\Omega (JN_i)\mathrm{d}x\mathrm{d}y\mathrm{d}t\ . \qquad (11)$$

V. Analysis Result of Eddy-Current Field

The iron-cored inductor shown in Fig. 3 is analyzed by the 3D space-time FEM, where the conductivity and relative permeability of iron core are 10^6 S/m and 5000, and the excitation frequency is 50 Hz. The natural boundary condition is imposed at the symmetric boundaries, and $A = 0$ at the outer boundaries.

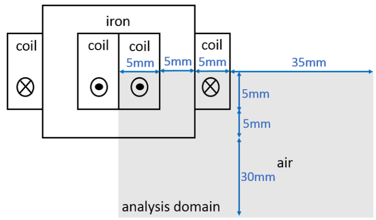

Fig.3 Iron-cored inductor

Fig. 4 shows magnetic flux lines obtained by the conventional FEM and by the space-time FEM using the triangular prism element. Both results agree. The space-time FE analysis of moving objects will be discussed in the full paper.

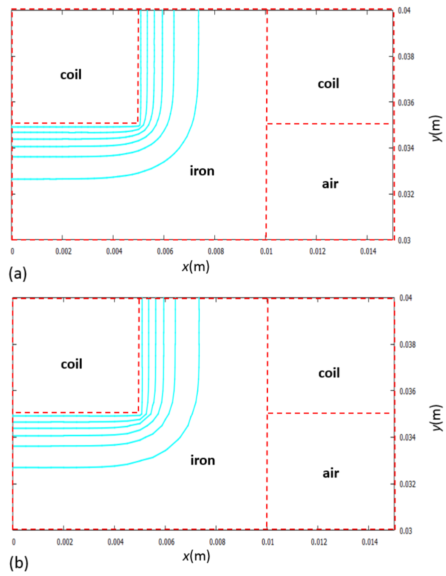

Fig.4 magnetic flux lines: (a) obtained by the conventional FEM, and (b) obtained by the space-time FEM using the triangular prism element

VI. References

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